

SOLVABILITY OF HORNSAT AND CNFSAT

KOBAYASHI, KOJI

1. OVERVIEW

This article describes the solvability of HornSAT and CNFSAT. Unsatisfiable HornCNF have partially ordered set that is made by causation of each clauses. In this partially ordered set, Truth value assignment that is false in each clauses become simply connected space. Therefore, if we reduce CNFSAT to HornSAT, we must make such partially ordered set in HornSAT. But CNFSAT have correlations of each clauses, the partially ordered set is not in polynomial size. Therefore, we cannot reduce CNFSAT to HornSAT in polynomial size.

2. CAUSATION OF HORNSAT

We show the simply connectivity of *HornSAT*. Because *HornSAT* is made by causation of each clauses, we can use unit resolution at *HornSAT*. Therefore, *HornSAT* have partially ordered set that become simply connected space.

Definition 1. In formula $F \in CNF$, we will use the term “Metric space of F ” as the metric space of truth value assignments that are false in F . In partially ordered set that element is subset of clauses in CNF, we think partially ordered set $\{f_i, <\}$ that different of successor is one clause;

$$f_p < f_q \rightarrow f_p \wedge c = f_q, f_p, f_q \subset F, c \in F$$

I will use the term “Normal series of F ” as $\{f_i, <\}$ if all f_i become simply connected space.

In addition, we show normal series as the partially ordered set of clauses of difference elements. That is, if normal series is $f_p < f_q < f_r$ and $f_q \setminus f_p = c_{pq}, f_r \setminus f_q = c_{qr}$, we show normal series as $c_{pq} < c_{qr}$.

Theorem 2. $F \in \overline{HornSAT}$ have normal series. Size of normal series at most F size.

Proof. We can use unit resolution at $\overline{HornSAT}$. Therefore, we can resolute each variables from each clauses by using unit resolution. Each element f_i of this partially ordered set become true in truth value assignment that all value are true, and become false in other truth value assignments. That is, all f_i become simply connected space. And size of this normal series is the size of unit resolution that necessary to resolute empty clause. Therefore, this theorem is proved. \square

3. HORNSAT AND CNFSAT

We show the *HornSAT* that include *CNFSAT*. Using this *HornSAT*, We can deal with *CNFSAT* in *HornSAT* structure.

Theorem 3. If *CNFSAT* is in P , polynomial size *HornSAT* that include *CNFSAT* is exists.

Proof. Think a series of computation that reduce $CNFSAT$ to $HornSAT$ and compute result. We can compute this computation with DTM. Therefore, $HornSAT$ is exist that emulate this DTM computation. Because This DTM computation include $CNFSAT$ problem as the start configuration, $HornSAT$ that emulate DTM computation include $CNFSAT$ problem. And if $CNFSAT$ is in P, DTM computation and $HornSAT$ that emulate DTM computation are also in P. Therefore, this theorem is proved. \square

4. CORRELATIONS OF CNFSAT

We show that reduction \overline{CNFSAT} to $\overline{HornSAT}$. Mentioned above 2, $\overline{HornSAT}$ have normal series. Therefore $\overline{HornSAT}$ that reduce \overline{CNFSAT} have normal series. But \overline{CNFSAT} have correlations of clauses, we cannot reduce \overline{CNFSAT} to $\overline{HornSAT}$ with polynomial size.

Theorem 4. *In $F \in \overline{CNFSAT}$, there are some pair of clauses $(c_{a0}, c_{a1}), (c_{b0}, c_{b1}), (c_{c0}, c_{c1}), \dots$ that are disconnected each other. f_a, f_b, f_c, \dots is separated space of $(c_{a0}, c_{a1}), (c_{b0}, c_{b1}), (c_{c0}, c_{c1}), \dots$. And $c_a, c_b, c_c, \dots, c_{ab}, c_{ac}, c_{bc}, \dots, c_{abc}, \dots$ is clauses that become false if and only if in some f_a, f_b, f_c, \dots power set $(f_a), (f_b), (f_c), \dots, (f_a \vee f_b), (f_b \vee f_c), (f_a \vee f_c), \dots, (f_a \vee f_b \vee f_c), \dots$ space.*

In this condition, normal series of F include partially ordered set $c_a, c_b, c_c, \dots, c_{ab}, c_{ac}, c_{bc}, \dots, c_{abc}, \dots$. That is;

$$\begin{aligned} & \vdots \\ & \dots c_{abc} < c_{ab} < c_a \\ & \dots c_{abc} < c_{ac} < c_a \\ & \dots c_{abc} < c_{ab} < c_b \\ & \dots c_{abc} < c_{bc} < c_b \\ & \dots c_{abc} < c_{bc} < c_c \\ & \dots c_{abc} < c_{ac} < c_c \\ & \vdots \end{aligned}$$

Proof. I prove it using reduction to absurdity. We assume that normal series of F is exist that not include partially ordered set $c_a, c_b, c_c, \dots, c_{ab}, c_{ac}, c_{bc}, \dots, c_{abc}, \dots$ in theorem condition.

We think the case that does not include c_{ab} in normal series. From the condition of theorem, the space exist that only c_{ab} become false. Therefore, if normal series do not include $c_{ab}, c_{ac} < c_a$ cannot keep normal series condition that c_{ac}, c_a difference is one clause and contradicts a condition. This condition are not only c_{ab} but also any other clauses.

Therefore, this theorem was shown than reduction to absurdity. \square

Theorem 5. *We can make some clauses that become false if and only if in some f_a, f_b, \dots power set $(f_a), (f_b), \dots, (f_a \vee f_b), \dots$ space in polynomial size of f_a, f_b, \dots .*

Proof. c_a, c_b, \dots is the clauses that become false if and only if in some $(f_a), (f_b), \dots$ space, and c_A, c_B, \dots is the clause that become true in all $(f_a), (f_b), \dots$ space. We can make proper CNF with parameter x_0, x_1, x_2, \dots ;

$$\begin{aligned} & (c_a \vee x_0) \wedge (c_A \vee x_0) \wedge (\overline{x_0} \vee c_b \vee x_1) \wedge (\overline{x_0} \vee c_B \vee x_1) \wedge \dots \\ & = (c_a \vee c_b \vee \dots) \wedge (c_A \vee c_b \vee \dots) \wedge (c_a \vee c_B \vee \dots) \wedge (c_A \vee c_B \vee \dots) \wedge \dots \end{aligned}$$

The CNF become false if and only if in $(f_a), (f_b), \dots, (f_a \vee f_b), \dots$ space. And this CNF is at most twice size of f_a, f_b, \dots . Therefore, this theorem is proved. \square

Theorem 6. *Normal series of some \overline{CNFSAT} problem become over polynomial size.*

Proof. Mentioned above 4, some \overline{CNFSAT} problem have normal series that include power set of pair of disconnection clauses. And mentioned above 5, we can make these problem at most constant size of pair of disconnection clauses. Therefore, this theorem is proved. \square

Theorem 7. $\overline{CNFSAT} \not\leq_p \overline{HornSAT}$

Proof. Mentioned above 6, some \overline{CNFSAT} problem have over polynomial size normal series. But mentioned above 3, if $CNFSAT$ is in P, we can make $HornSAT$ that include $CNFSAT$ problem in polynomial size. And mentioned above 2, size of normal series of $\overline{HornSAT}$ is at most formula size and contradicts a condition that $CNFSAT$ is in P.

Therefore, we cannot \overline{CNFSAT} to $\overline{HornSAT}$ in polynomial size. \square

REFERENCES

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